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**THE FIREPOWER OF A  
SURFACE-TO-AIR MISSILE  
SYSTEM AGAINST CROSSING RAIDS**

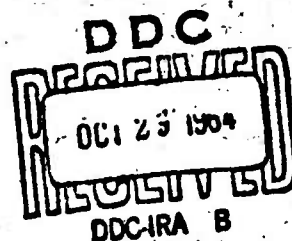
By N. Timenes

Research Contribution No. 58

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30 July 1964

RESEARCH CONTRIBUTION  
**Operations Evaluation Group**

CENTER FOR NAVAL ANALYSES

THE FIREPOWER OF A  
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#### ABSTRACT

Previously published graphic and algebraic methods of computing the maximum number of salvos a surface-to-air missile (SAM) battery can fire against a crossing raid are reviewed. Examination of the nature of an algebraic approximation shows that difficulties arise when considering targets which reach their bomb release point after the point of closest approach to the SAM battery, or which have low velocity with respect to SAM velocity. An alternative graphic method solution is suggested, which facilitates locating intercept points when investigating an engagement between a multi-channel SAM battery and a multiple-target raid.



## I. INTRODUCTION

Reference (a) presents several convenient and widely used methods for estimating the firepower of a surface-to-air missile (SAM) battery against air targets. Comments are offered here on two of these: one algebraic and one graphic. Reference (a) develops formulas for calculating the maximum possible number of missile salvos a SAM battery could fire against a single target (or point raid), for launcher-limited and for guidance-limited cases. The solution given for the launcher-limited case has general applicability. On the other hand, the formula for the case in which the SAM ship is guidance-limited (or limited by a "shoot-look-shoot" firing doctrine) is exact only for the direct approach raid. In other cases, an approximation is necessary. Difficulty in choosing a suitable approximation can occur when the SAM ship lies well off the raid track, when the raid velocity is low with respect to SAM velocity, or when the target aircraft reaches its bomb release point after its point of closest approach to the SAM battery. The nature of these difficulties is discussed in section II, below.

Reference (a) also suggests the possibility of graphic solution for crossing raids, using a curved track for the SAM in the  $x - T$  plane (p.88), where  $T$  is the intercept interval (the time between intercepts) and  $x$  is measured along the raid track. However, use of the  $x - T$  plane prevents full exploitation of the capabilities of the graphic method when considering multiple targets arriving in arbitrary formation. Plotting range from the SAM battery to the target against time, measured over the entire engagement, facilitates analysis in more complex cases, and is discussed in section III.

## II. ALGEBRAIC SOLUTION: AN APPROXIMATION

In the discussion which follows, it is assumed that the constraints of the SAM system against targets of interest will determine nominal or expected values for maximum and minimum engagement ranges, the time of first possible missile launch and of last possible intercept, launcher cycle and guidance-channel switching times, and the speed of the missile.

Let the time between successive intercepts (the "intercept interval") be  $T = \alpha + \beta p$ , where  $\alpha$  and  $\beta$  are constants characteristic of the SAM system, and  $p$  is range from the SAM battery at intercept (figure 1). In the guidance-limited or "shoot-look-shoot" case,  $\beta \neq 0$ , and the intercept interval ( $T$ ) is equal to flight time of the missile plus a guidance-channel delay time, or an evaluation time, which may be incorporated in  $\alpha$ .

Let  $x_1$  be the position of the  $i^{\text{th}}$  intercept on the target track,  $u$  the target speed, and  $(x_0, y_0)$  the coordinates of the point of closest approach to the SAM ship. Then,



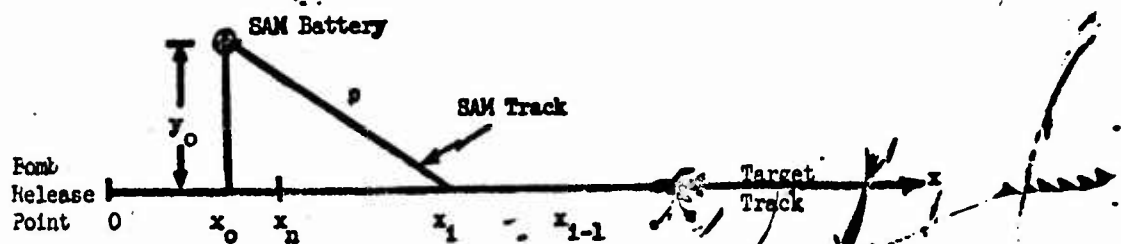


FIG.1: ENGAGEMENT GEOMETRY IN REAL SPACE



$$T = \frac{x_{1-1} - x_1}{u}$$

$$= \alpha + \beta \rho.$$

From figure 1, it is clear that

$$T = \alpha + \beta \sqrt{(x_1 - x_0)^2 + y_0^2}. \quad (1)$$

In case the SAM battery lies on the raid axis,  $y_0 = 0$ , and

$$T = \alpha + \beta(x_1 - x_0). \quad (2)$$

Using this linear relation, reference (a) derives a formula for  $n$ , the maximum number of missile salvos which can be fired against this target:

$$n = \left[ 1 + \frac{\log \left( \frac{\alpha + \beta x_1}{\alpha + \beta x_n} \right)}{\log(1 + u\beta)} \right], \quad (3)$$

where  $[z]$  denotes the greatest integer not exceeding  $z$ , and  $x_1$  and  $x_n$  are the x-coordinates of the first and last possible intercepts respectively. This formula is exact if the raid approaches the SAM ship directly. However, if the SAM unit is located off the raid axis, use of this formula requires choosing an approximation,  $T'$ , for  $T$  which is linear in  $x_1$ :

$$T' = \alpha' + \beta' x_1$$

$$\approx \alpha + \beta \sqrt{(x_1 - x_0)^2 + y_0^2}. \quad (4)$$

For suitable choices of  $\alpha'$  and  $\beta'$ , then, reference (a) develops the following approximation to  $n$ :

$$n' = \left[ 1 + \frac{\log \left( \frac{\alpha' + \beta' x_1}{\alpha' + \beta' x_n} \right)}{\log(1 + u\beta')} \right]. \quad (5)$$



Consider now the circumstances under which such an approximation is valid. Equation (4) may be rewritten, if  $x_1 \neq x_0$ , as:

$$\alpha' + \beta'x_1 \approx \alpha + \beta(x_1 - x_0) \left[ 1 + \left( \frac{y_0}{x_1 - x_0} \right)^2 \right]^{\frac{1}{2}} \quad (6)$$

$y_0 \ll (x_1 - x_0)$ , letting  $\alpha' = \alpha - \beta x_0$  and  $\beta' = \beta$  will provide a good approximation. Then,

$$T' = \alpha + \beta(x_1 - x_0). \quad (7)$$

If, however,  $(x_1 - x_0) \sim y_0$ , this approximation is unattractive. The reason for this difficulty becomes apparent upon examination of the nature of the function approximated. Rewriting equation (1),

$$\left( \frac{T - \alpha}{\beta y_0} \right)^2 - \left( \frac{x_1 - x_0}{y_0} \right)^2 = 1. \quad (8)$$

This is a hyperbola in the  $x_1 - T$  plane, with asymptotes

$$T = \alpha \pm \beta(x_1 - x_0). \quad (9)$$

Thus, equation (7) approximates a hyperbola by one of its asymptotes. Figure 2 shows a graph of equation (8) with its asymptotes (equations (9)). Clearly, approximation of a branch of this hyperbola by one of its asymptotes (or by any other linear function) breaks down for values of  $x$  near  $x_0$ . Large errors can occur if  $x_1 < x_0$ ; hence, the formula should not be used against targets for which  $x_n < x_0$  (or, for which the "bomb release point" is reached after the point of closest approach to the SAM battery).

The usefulness of an approximation depends on the effect on  $n$  of errors in  $T$ . The difference between the actual value of  $n$  and the value indicated by the approximation will depend on the flatness of the hyperbola in the area of interest. Figure 3 illustrates a situation in which using the asymptote as an approximation would result in an error in  $n$ . Choosing a more appropriate linear approximation presupposes considerable advance knowledge. Furthermore, different approximations may be required for small changes in the tactical situation, such as a change in the bomb release line. These difficulties arise if the portion of the hyperbola under consideration deviates significantly from the linear. If the SAM ship lies



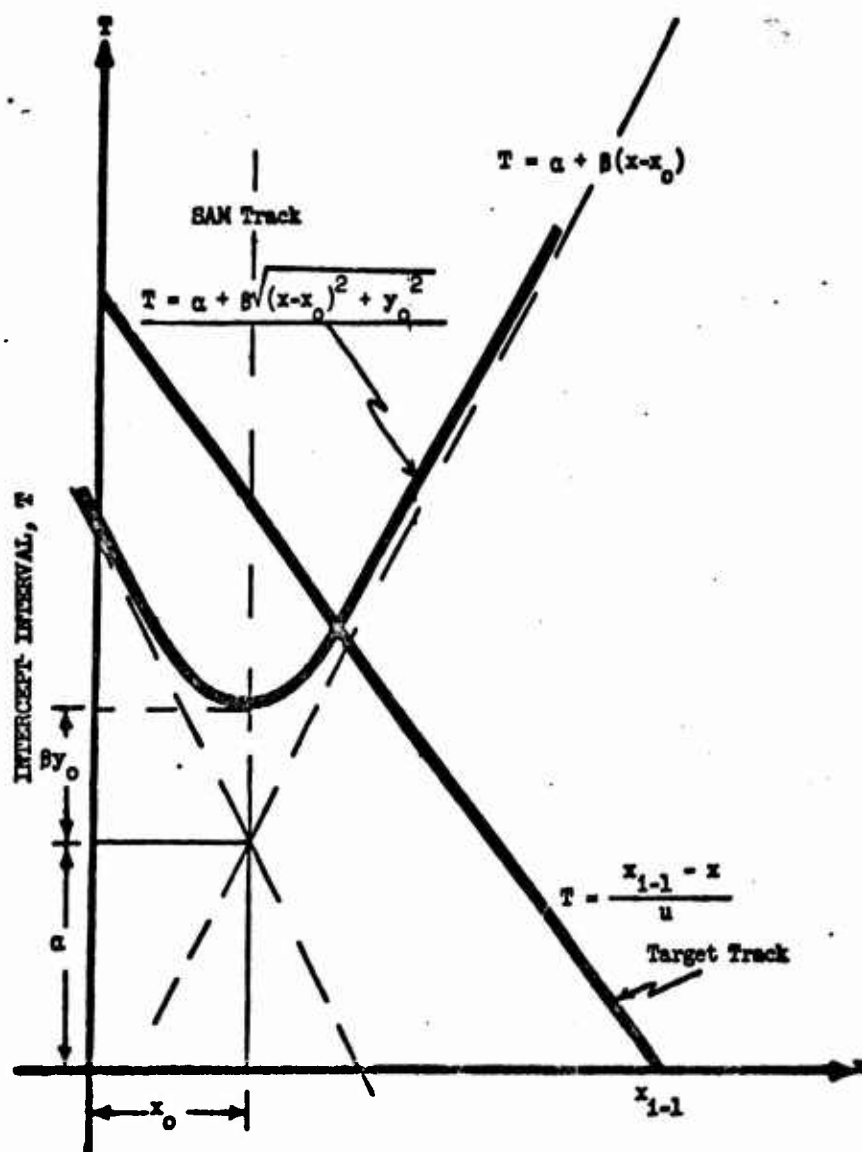


FIG. 2: ENGAGEMENT GEOMETRY IN  $x - T$  SPACE



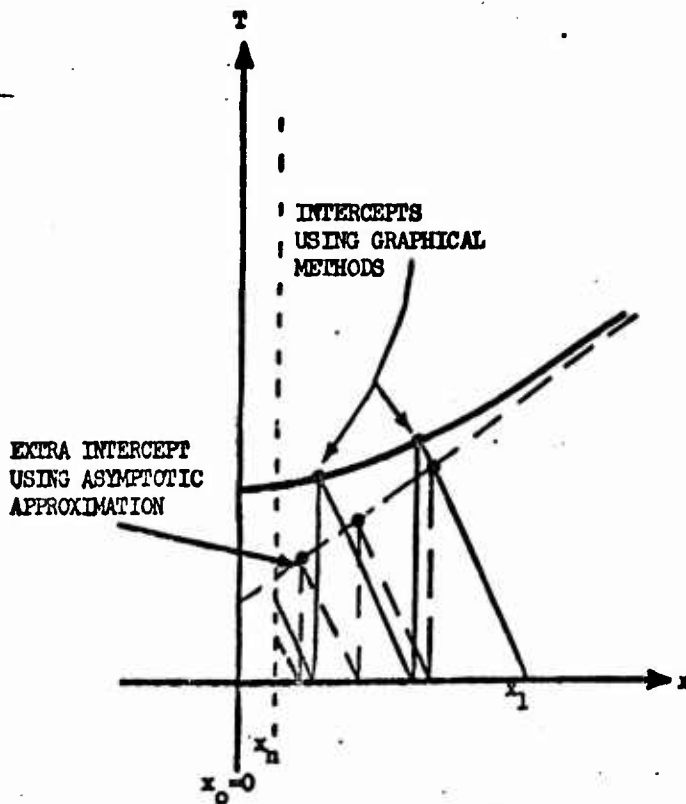


FIG. 3: POSSIBLE INTRODUCTION OF ERROR IN  $n$



well off the raid track ( $y_0$  large), the maximum difference between the hyperbola and its nearest asymptote,

$$T - T' = 2y_0 \quad (10)$$

(which occurs when  $x = x_0$ ), can be large with respect to the geometry of interest, and hence cause difficulty in choosing a good approximation. Further, the possibility of an error in  $n$  is increased if the slope of the line representing the target track in the  $x - T$  plane is steep. The equation of this line is:

$$T = \frac{x_{i-1} - x_i}{u}, \quad (11)$$

and the slope ( $\frac{1}{u}$ ) is the inverse of target velocity. Hence, care must also be exercised in selecting an approximation for use against slow-speed targets.

### III. GRAPHIC SOLUTION

These difficulties can be avoided by solving for  $n$  graphically. While the graphic solution using the  $x_i - T$  plane suggested in reference (a) is clearly possible, it becomes quite complicated if an engagement involves more than one SAM launcher, one guidance system, and one target. Further, two targets flying parallel tracks (with different values of  $y_0$ ) will require different SAM hyperbolas. A more convenient method of graphic solution, using one graph for an entire engagement, is based on the following observations.

Consider a single time scale ( $t$ ) on which the entire history of an engagement may be recorded (rather than just the intercept interval ( $T$ )). Taking  $t = 0$  when  $\rho = y_0$ , the target track is represented by  $x - x_0 = ut$ , and, from figure 1,

$$\begin{aligned} \rho^2 &= y_0^2 + (x - x_0)^2 \\ &= y_0^2 + (ut)^2 \end{aligned} \quad (12)$$

This is the equation of a hyperbola in the  $t - \rho$  plane. Only the branch of the hyperbola for which  $\rho$  is positive is of interest (figure 4). If the raid passes over the SAM ship,  $y_0 = 0$ , and (12) reduces to



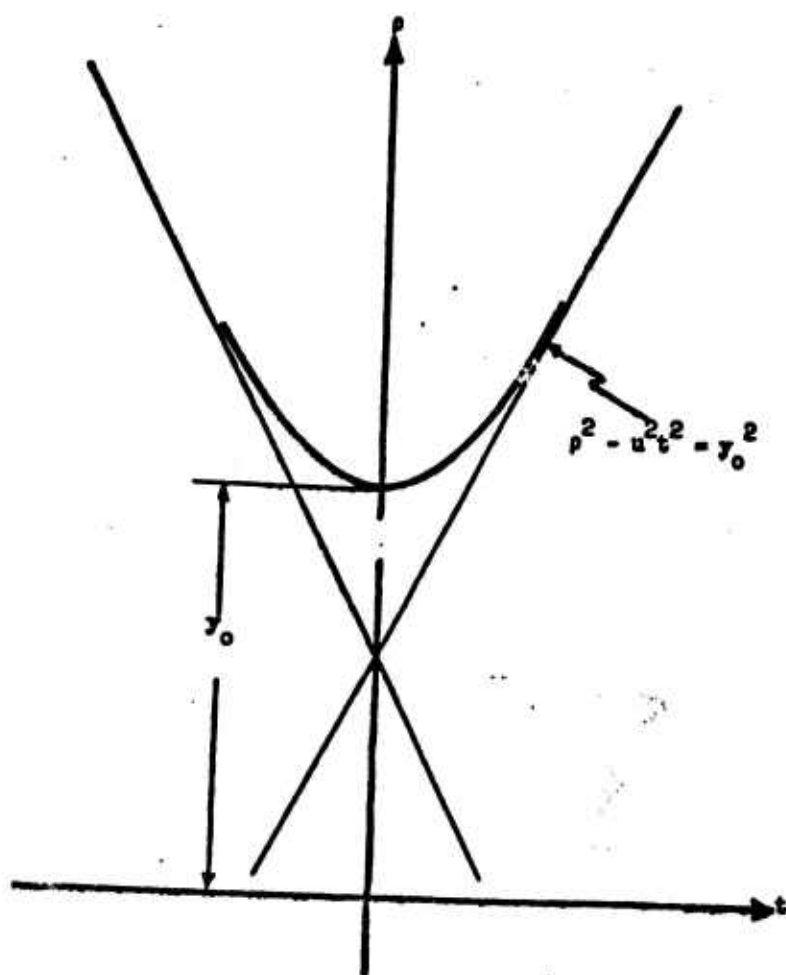


FIG. 4: TARGET TRACK IN THE  $t$  -  $p$  PLANE



$$\rho^2 = (ut)^2,$$

which gives the asymptotes of the hyperbolas for which  $y_0 \neq 0$ :

$$\rho = \pm ut.$$

Suppose several aircraft attack at the same speed ( $u$ ). If they fly abreast (wave attack), they will differ only in their crossing distances,  $y_0$ , and their tracks will form a family of hyperbolas in the  $t - \rho$  plane, with a common major axis and common asymptotes (figure 5). Similarly, in a stream attack, in which the intruders follow one another at time intervals  $\Delta t$ , the hyperbolas are of identical shape with their major axes parallel and  $\Delta t$  apart on the time scale (figure 6). More complex formations are represented by combinations of these cases.

If  $t_1$  is the time of launch of the  $i^{\text{th}}$  missile, and the time of flight is  $\alpha + \beta\rho$ , then

$$T = \alpha + \beta\rho$$

$$= t - t_1$$

is a straight line in the  $t - \rho$  plane. The SAM salvo track, in the  $t - \rho$  plane, is the segment of this line originating on the  $t$  - axis and terminating upon intersection with the target track. The time ( $t_1$ ) of first

SAM salvo launch is determined by such factors as radar detection range, information processing times, and maximum missile range. The slope of the SAM track is determined only by SAM speed. The spacing of successive SAM salvo tracks along the  $t$  - axis is determined by launcher reload time,  $T_L$ , or guidance system delay times,  $T_G$ . A family of such segments representing missile salvos may be compared with a hyperbolic branch or a family of hyperbolic branches representing raid tracks (figure 7). The maximum number ( $n$ ) of possible missile salvos is obtained by counting the number of SAM salvo track-target track intersections.

Note that the single-target, launcher-limited or guidance-limited engagement is a special case of the duel between a SAM ship and a hostile raid. The most general case consists of multiple targets, flying different tracks, possibly with different velocities, combined with both launcher and guidance-channel limitations. Typically, guidance channel availability will govern firepower against targets at long ranges, while launcher cycle times will limit the number of salvos against targets at short ranges (for which missile flight times are short). When the limiting factor



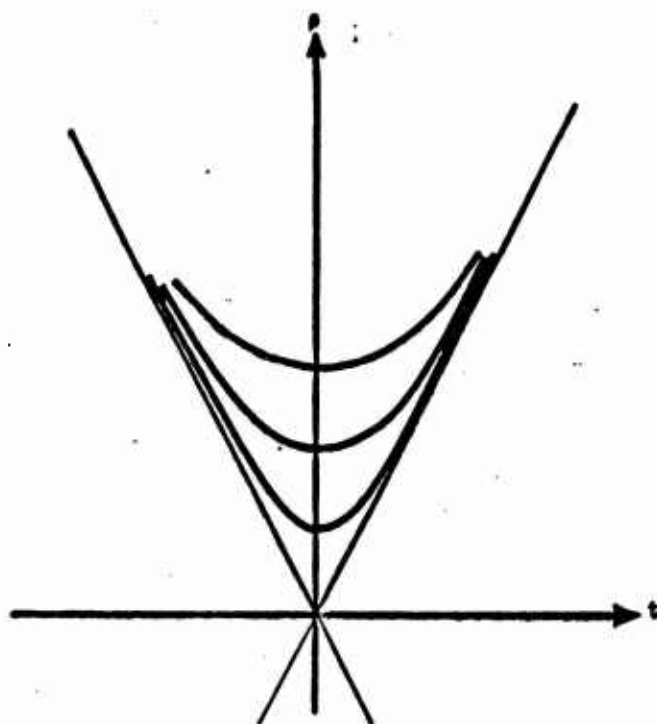


FIG. 5: TARGET TRACKS - WAVE ATTACK

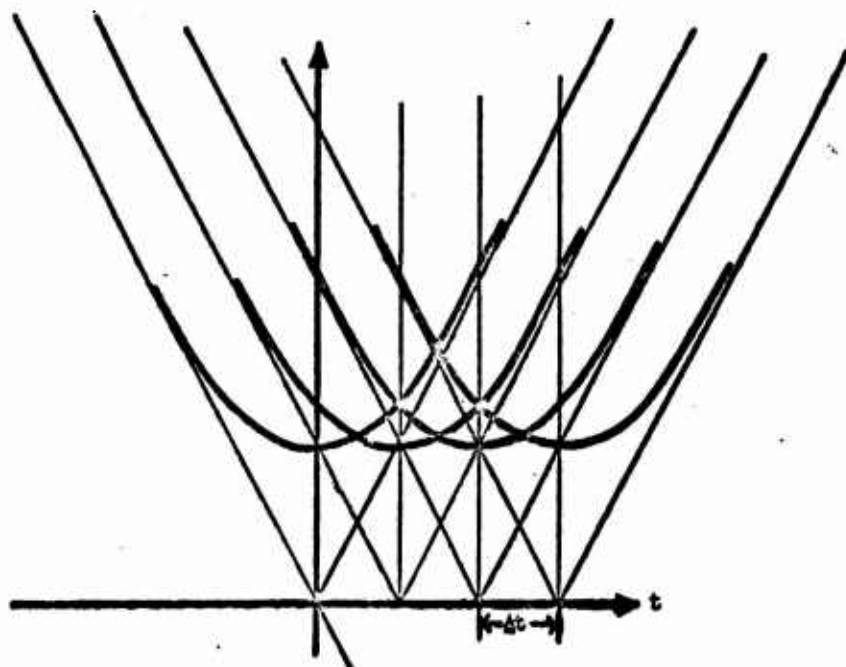


FIG. 6: TARGET TRACKS - STREAM ATTACK



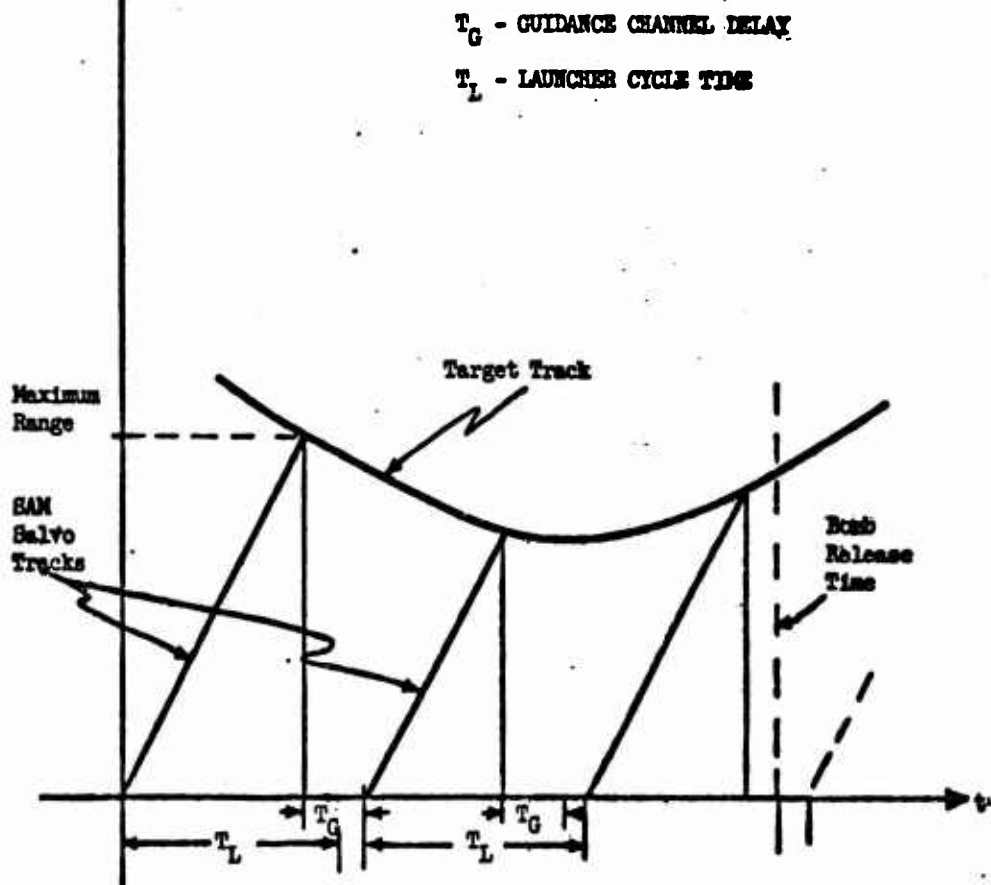


FIG. 7: GRAPHIC SOLUTION IN  $t - p$  PLANE



changes from guidance availability to launcher cycle time during an engagement, the formulas developed in reference (a) are not applicable. A graphic solution, such as that shown in figure 7, is possible in all cases. The writer has found it convenient to draw missile tracks on a transparent overlay, and to draw families of raid tracks on graph paper. This avoids the necessity of redrawing raid curves for use with different SAM systems, and permits rapid examination of the effects of variations in raid tactics, missile firing doctrine, and equipment.



Reference: (s) M. C. Waddell, "Surface-to-Air Guided Missile Systems:  
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